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A DETAILED GRAVIMETRIC GEOID OF NORTH AMERICA, THE NORTH ATLANTIC, EURASIA, AND AUSTRALIA*

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ABSTRACT

A detailed gravimetric geoid of North America, the North Atlantic, Eurasia, and Australia computed from a combination of satellite-derived and surface $1^\circ \times 1^\circ$ gravity data, is presented herein. Using a consistent set of parameters, this geoid is referenced to an absolute datum. The precision of this detailed geoid is ±2 meters in the continents but may be in the range of 5 to 7 meters in those areas where data was sparse. Comparisons of the detailed gravimetric geoid with results of Rice for the United States, Bomford and Fischer in Eurasia, and Mather in Australia are presented. Comparisons are also presented with geoid heights from satellite solutions for geocentric station coordinates in North America, the Caribbean, Europe, and Australia.

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1. INTRODUCTION

This paper presents a detailed gravimetric geoid of North America, the North Atlantic, Eurasia and Australia based upon a combination of satellite derived and surface 1° x 1° gravity data. Early gravimetric geoid computations were based upon surface gravity data; Hirvonen (1934) and Tanni (1948, 1949). The most ambitious of the pre-satellite gravimetric geoids was the Columbus geoid (Heiskanen, 1957). All of these pre-satellite geoids suffered from a lack of worldwide gravity coverage. With the advent of satellites it has been possible to derive the long wavelength components of the gravity field on a worldwide basis with considerable accuracy. The satellite gravity data can be combined with surface gravity data, in areas where surface gravity is available, to provide accurate estimates of the details of the geoidal undulations. This ability to combine the two-data types to obtain detailed geoid undulations in local areas through combination of surface and satellite gravity was recognized early in satellite geodesy (Khan and Strange, 1966) and awaited only the gathering of sufficiently accurate data. The method of Khan and Strange (1966) has been applied essentially unchanged to derive the results presented in this paper.

The geoid is becoming increasingly important for the support of research in geodesy and geophysics. The Skylab and GEOS-C spacecraft will carry radar altimeters for the purpose of measuring the geoid undulations in oceanic areas. An independently derived geoid map will provide a valuable complement to these experiments. By studying the gravimetric geoid, optimum experiment locations can be established. Also, the gravimetric geoid can be used to calibrate and check the accuracy of the altimeter. An accurate geoid map is also valuable for satellite and inertial navigation systems which are being used for offshore mineral exploration. A number of experimenters have derived values for tracking station coordinates from satellite observations. For the cases where the height

of the station above mean sea level is known accurately, the geoid map can be used to check the accuracy of the derived height above the ellipsoid in past solutions. Furthermore, the detailed geoid can be used as a constraint for future solutions as was recently done by Mueller and Whiting, 1972 who incorporated an earlier GSFC detailed gravimetric geoid map (Vincent et al., 1971) into their global geometric solution. Over the continents the gravimetric geoid can be compared with astrogeodetic geoids. These comparisons provide not only an indication of the relative accuracy of the geoids but they also provide information on tilts of the major geodetic datums with respect to a geocentric reference system.

In previous publications (Strange, et al., 1971; Vincent, et al., 1971) detailed geoid height maps were presented covering a substantial part of the northern hemisphere. As was stated in the previous papers the geoid heights given were only accurate to within an additive constant. This constant difference between the geoid heights presented and true geoid heights included a zero order undulation error (Rapp, 1967) plus other small systematic errors. Since the presentation of these previous geoids, a study has been made to determine best estimates of the various parameters which could cause the systematic errors (Strange and Richardson, 1972). Using a consistent set of parameters based on the results of this study it is now possible to remove the systematic error to a large extent. The objective of this paper is to present a set of detailed gravimetric geoid height maps which are based on a consistent set of parameters

that are very nearly true values for the earth. Systematic errors in the geocentric radius vectors to the geoid computed using the geoid heights of these maps and a reference ellipsoid of $a_e = 6378.142$ kms and $f = 1/298.258$ are almost certainly less than 5 meters, i.e. an accuracy for the radius vector of better than 1 ppm.

In addition to being referenced to an absolute datum, the maps presented here also differ from those previously published in that additional observational data has been used and the area covered in previous reports has been increased to include Australia and Canada.

The detailed gravimetric geoids presented here have a precision of ±2 m rms. This precision was established by comparing the detailed gravimetric geoid with Rice's astrogeodetic geoid for the United States, Bomford's astrogeodetic geoid for Europe and Mather's astrogeodetic geoid for Australia.

Comparisons have also been made between the detailed gravimetric geoid and satellite derived tracking station positions of Goddard Space Flight Center (GSFC) and Smithsonian Astrophysical Observatory (SAO).

2. METHOD OF COMPUTATION

The geoidal undulation at any point P on the earth can be computed using the well known Stokes' formula:

$$N(\varphi, \lambda) = \frac{R}{4\pi G} \int_{\lambda'=0}^{2\pi} \int_{\varphi'=-\frac{\pi}{2}}^{\pi/2} \Delta g_T(\varphi', \lambda')\; S(\psi) \cos \varphi' d\varphi' d\lambda' \tag{1}$$

where:

φ, λ = The geocentric latitude and longitude, respectively, of the computation point.

φ', λ' = The geocentric latitude and longitude, respectively, of the variable integration point.

$N(\varphi, \lambda)$ = Geoid undulation at φ, λ .

R = Mean radius of the earth.

G = Mean value of gravity over the earth

$\Delta g_T(\varphi', \lambda')$ = Free air gravity anomaly at the variable point φ', λ'.

$$S(\Psi) = \frac{1}{\sin(\Psi/2)} - 6 \sin(\Psi/2) + 1 + 5 \cos \Psi$$

$$- 3 \cos \Psi \ \ln(\sin (\Psi/2) + \sin^2 (\Psi/2))$$

where

$$\Psi = \cos^{-1}[\sin \varphi \sin \varphi' + \cos \varphi \cos \varphi' \cos(\lambda - \lambda')]$$

In order to combine surface and satellite gravity data for geoid computation, the earth is divided into two areas, a local area (A_1) surrrounding the point P, and the remainder of the earth (A_2). Also the anomalous gravity in each area is partitioned into two parts represented by the symbols Δg_s and Δg_2. The Δg_s values are defined as that part of the anomalous gravity field which can be represented by the coefficients in a satellite derived spherical harmonic expansion of the gravitation potential. The 1969 SAO Standard Earth (Gaposchkin and Lambeck, 1970) was used in all computations described in this paper. The Δg_2 values are defined as the remainder of the anomalous gravity field. Using this division of the earth's surface into two areas and of the anomalous gravity into two components one can write equation (1) in the form:

$$N(\varphi, \lambda) = N_1 + N_2 + N_3 \tag{2}$$

where

$$N_1 = \frac{R}{4\pi G} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\Delta g_s(\varphi', \lambda') \, S(\Psi) \cos\varphi' \, d\varphi' \, d\lambda' \right]$$

$$N_2 = \frac{R}{4\pi G} \iint_{A_1} \left[\Delta g_2(\varphi', \lambda') \, S(\Psi) \cos\varphi' \, d\varphi' \, d\lambda' \right] \tag{3}$$

$$N_3 = \frac{R}{4\pi G} \iint_{A_2} \left[\Delta g_2(\varphi', \lambda') \, S(\Psi) \cos\varphi' \, d\varphi' \, d\lambda' \right]$$

The following paragraphs discuss how each of the three components presented in equation (3) is handled in the computations.

Given a set of satellite derived coefficients in the spherical harmonic expansion of the gravitational potential, a number of methods exist for the computation of the N_1 component of the geoid undulation.

The computation of N_1 was not carried out in the present case by using the integration indicated in equation (3). Rather the procedure described by Bacon, et al., (1970), was used. Briefly this procedure consists of fixing a value of the potential, W_o, and computing the component N_1 as

$$N_1 = r - r_E \tag{4}$$

where:

r = is the radial distance to the equipotential surface defined by W_o and the potential coefficients of the SAO 1969 Standard Earth.

r_E = is the radial distance to a selected reference ellipsoid defined by a semimajor axis (a_e) and flattening (f).

The radial distance, R_G, to the equipotential surface W_o at a particular latitude and longitude ϕ_1, λ_1 is determined by using the equation

$$W_0 = \psi(r,\phi,\lambda) = \frac{kM}{r}\left[1 - \sum_{n=2}^{k}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n (\bar{C}_{nm}\cos m\lambda_1 + \bar{S}_{nm}\sin m\lambda_1)\, P_{nm}(\sin\phi)\right]$$

$$+ \frac{\omega^2 r^2}{2}\cos^2\varphi \tag{5}$$

where

kM = the product of the gravitational constant and the mass of the earth

a_e = semimajor axis of the reference ellipsoid

r = geocentric radius

ω = earth's angular velocity

$\bar{C}_{nm}$ and $\bar{S}_{nm}$ = fully normalized spherical harmonic coefficients of the gravitational potential

$P_{nm}(\sin\phi)$ = the Associated Legendre Polynomial.

The only unknown in this equation is r. Values of $r_1 = R + \epsilon$, $r_2 = R - \epsilon$, and $r_3 = (r_1 + r_2)/2$ are chosen for substitution into equation (5) for evaluation of the functions

$$\psi_1(r_1,\phi,\lambda), \psi_2(r_2,\phi,\lambda), \text{and } \psi_3(r_3,\phi,\lambda).$$

The r_i for which $|\psi_i - W_o|$ is a maximum is identified and eliminated from consideration. The two remaining values of r_i are labeled r_1 and r_2 and are

used for calculation of $r_3 = (r_1 + r_2)/2$. The potential functions are evaluated with these arguments and the worse-value elimination process is repeated. The process continues until an r is chosen such that $|\psi(r, \phi, \lambda) - W_o| \leq 10^{-12}$. Using this value of r and the value of r_E computed using the input values of a_e and f of the reference ellipsoid, a geoid undulation component N_1 is computed.

For the computations described in this paper, the area A_1 for a point at which the geoid was being computed was defined to consist of a twenty degree by twenty degree area centered on the computation point. The computational formula used was:

$$N_2 = \frac{R}{4\pi G} \sum_{j=1}^{400} \overline{\Delta g}_2 (\varphi_j', \lambda_j') \, S(\Psi_j) \cos \varphi_j' \, \Delta\varphi' \Delta\lambda' \tag{6}$$

where

$\overline{\Delta g}_2(\varphi_j', \lambda_j')$ is the mean value of Δg_2 within the j^{th} $1^\circ \times 1^\circ$ square

$S(\Psi_j)$ is the value of Stokes' function at the center of the j^{th} $1^\circ \times 1^\circ$ square.

$\Delta\varphi' = \Delta\lambda' = 1^\circ$.

The value of $\overline{\Delta g}_2$ used for each $1^\circ \times 1^\circ$ square was computed using the formula

$$\overline{\Delta g}_2 = \overline{\Delta g}_e - \overline{\Delta g}_s \tag{6}$$

The $\overline{\Delta g}_e$ values are mean $1^\circ \times 1^\circ$ free-air anomalies provided by surface gravity data. The primary source material for this gravity data is as follows:

1) North America: Strange and Woollard (1964), Woollard (1968), Nagy (1970) and ACIC (1971)

2) North Atlantic: Bowin (1971), Talwani (1971), Strang Van Hees (1970), and ACIC (1971)

3) Eurasia: Tengström (1965), Arnold (1964). Bowin (1971), and ACIC (1971)

4) Australia: Mather (1969).

Values of Δg_e for each $1° \times 1°$ square were computed by carrying out the computation

$$\overline{\Delta g}_e = \overline{\Delta g}_{IF} + \gamma_{IF} - P.C. - \gamma_N$$

where

$\overline{\Delta g}_{IF}$ = mean value of free air anomaly referred to the international gravity formula

γ_{IF} = international gravity formula

P.C. = Potsdam correction with a value of -13.7 mgals

γ_N = $978.0322\,(1+.0053025 \sin^2 \varphi - .00000585 \sin^2 2\varphi)$ gals

In carrying out the computations γ_{IF} and γ_N were evaluated at the center of each $1° \times 1°$ square.

The $\overline{\Delta g}_S$ values are that part of the mean $1° \times 1°$ free-air anomalies represented by the satellite harmonic coefficients used in computing N_1. The $\overline{\Delta g}_s$ values are obtained by evaluating the following equation at the center of each $1° \times 1°$ square.

$$\overline{\Delta g}_s = \gamma_e \sum_{n=2}^{k} \sum_{m=0}^{n} (n-1) \left[\overline{C}_{nm} \cos m\lambda' + \overline{S}_{nm} \sin m\lambda' \right] P_{nm}(\sin \phi') \qquad (7)$$

where

γ_e = Equatorial gravity in milligals

k = Upper limit on degree and order of the geopotential model

n = Degree index of harmonic coefficients

m = Order index of harmonic coefficients

In equation (7), the $\bar{C}_{20}$ and $\bar{C}_{40}$ terms do not represent the complete coefficients but rather the difference between the complete coefficients and the coefficients compatible with the ellipsoid used in computing N_1. The difference values used were $\overline{\Delta C}_{20} = .03577 \times 10^{-6}$ and $\overline{\Delta C}_{40} = -.232 \times 10^{-6}$ (fully normalized). In order for the above described procedure to produce correct results, the quantities $\overline{\Delta g}_e$, $\overline{\Delta g}_s$, and the a and f which define the ellipsoid used to compute N_1 must all be compatible. Compatibility implies that the values of $\bar{C}_{20}$ and $\bar{C}_{40}$ used to compute the values of theoretical gravity needed to obtain $\overline{\Delta g}_e$ and $\overline{\Delta g}_s$ are the same as the values of $\bar{C}_{20}$ and $\bar{C}_{40}$ implied by the reference ellipsoid. Correct results in the absolute sense are also dependent upon the value of W_o chosen to represent the true value of the potential of the geoid. The effects of not making $\overline{\Delta g}_e$, $\overline{\Delta g}_s$, a, and f compatible are twofold. First, all the computed geoid heights may by in error by a constant; in addition, there will be a systematic error as a function of latitude. The effect of selecting an incorrect value of W_o would be to introduce a constant error in all geoid heights.

In the calculations described here the term N_3 in equation (2) is set equal to zero. this is equivalent to assuming that the satellite derived approximation to the gravity field is adequate for the area A_2 at a distance of greater than ten degrees from the computation point.

3. GEOIDAL SCALE

With the procedure described in Section 2, one can compute values of geoid height, N, which are accurate to within an additive constant provided that one; 1) uses the correct value of rotation rate, ω, 2) assumes that the reference gravity formula used in computing the surface gravity anomalies is compatible with the flattening chosen for the reference ellipsoid selected and 3) makes the $\Delta \bar{C}_{20}$ and $\Delta\bar{C}_{40}$ used in deriving the satellite component of geoid undulation compatible with the selected reference ellipsoid flattening. With these conditions satisfied, one need only use reasonable approximations to such parameters as W_o, kM, Potsdam correction, a_e, and γ_e to obtain values of N accurate to within an additive constant.

It is desirable, however, to produce geoid heights which do not have a constant uncertainty and that the parameters of the best fitting reference ellipsoid be known. The best fitting reference ellipsoid is defined here as that ellipsoid having semimajor axis, a_e, and flattening, f, such that its surface is an equipotential having the same value of the potential as the geoid, when the potential is computed using the kM, ω and J_2 of the actual earth. As a result of recent work (Strange and Richardson, 1972) it now appears possible to present absolute geoid heights and reference ellipsoid parameters with reasonable assurance that any systematic errors in geocentric radius vector to the geoid are less than 5 meters and probably of the order of 3 meters.

In using the theory described in Section 2. systematic errors can occur due to errors in the following parameters: kM, ω, J_2, a_e, f, γ_e, and the Potsdam correction. Strange and Richardson (1972) adopted values of

$$kM_{a+e} = 3.986012 \times 10^{20}/cm^3/sec^2$$

$$kM_e = 3.986009 \times 10^{20}\ cm^3/sec^2$$

$$\omega = .72921151467 \times 10^{-4}\ rad/sec$$

$$J_2 = 1082.6392 \times 10^{-6}$$

$$1/f = 298.258$$

Potsdam correction = -13.7 mgals

taken from previous work together with available surface gravity values to compute

$$\gamma_e = 978.0332\ gals$$

$$a_e = 6378.1388\ kms$$

$$W_o = 6263691.0\ kgal\ m$$

where

M_e = mass of the earth excluding the atmosphere

M_{a+e} = mass of the earth including the atmosphere

It was also found by comparison of detailed gravimetric geoid heights with dynamic station positions, that the values (after correcting the dynamic station positions to be compatible with the chosen values of kM_{a+e}) of a_e ranged from 6378.141 kms to 6378.144 kms. If these values were correct, commensurate changes in γ_e and W_o would be required to maintain a consistent set of parameters. Since the value of γ_e = 978.0332 gals obtained from surface gravity analysis could easily be in error by ±1.0 mgals, it was decided to adopt as the set of parameters for the geoid undulation the $k\,M_{a+e}$, kM_e, ω, J_2 and 1/f given above together with

$$\gamma_e = 978.0322 \text{ gals}$$

$$a_e = 6378.142 \text{ kms}$$

$$W_o = 6263687.5 \text{ kgal m}$$

Geoid undulations were then computed using this coherent set of parameters.

As a means of evaluating the scale of the geoid, detailed geoid heights and reference ellipsoid parameters were used together with mean sea level heights taken from the NASA Directory of Observation Station Locations (NASA, 1971) to compute geocentric radius vectors for a substantial number of satellite tracking stations. These geocentric radius vectors were then compared with the geocentric radius vectors derived from satellite observations by a number of investigators in order to determine the extent of systematic differences (tables 1 through 3). Since the scale of the dynamic orbit analysis is set by the value of $k\,M_{a+e}$ used, the dynamically derived radius vectors were modified to be compatible with $k\,M_{a+e} = 3.986012 \times 10^{20}\ \text{cm}^3/\text{sec}^2$ before making any comparisons. The dynamic radius vectors and those obtained using the gravimetrically derived parameters have a systematic mean difference of 3 meters or less. This level of agreement must be considered excellent taking into account the potential uncertainties in the various data used in deriving the computational parameters. Of the various potential sources of the differences, the most probable causes are:

1) Errors in detailed gravimetric geoid height at tracking stations due to the use of simple free air anomalies rather than terrain corrected free air anomalies.

2) Errors in values of γ_e, W_o and a_e

3) Errors in dynamic station coordinates including small deviations from the center of mass of the earth of the origin of the coordinate system to which dynamic station positions are referenced.

4) Errors in mean sea level elevations for some tracking stations.

Theoretically terrain corrected free-air anomalies rather than simple free air anomalies provide more accurate estimates of geoid height. The effect of using simple free air anomalies is to produce geoid heights which are systematically too negative in the vicinity of land areas with rugged relief. Dimitrijevich (1972) has shown that the value of the difference in the United States ranges from in excess of +3.5 meters in the rugged mountains of the western United States to about +0.2 meters along the east of the United States. Since most tracking stations used in the comparisons are on large land masses and several are in areas of rugged relief, one to two meters of the systematic difference can be assumed to arise from this source. It should be noted that differences due to this source are not the result of errors in basic parameters but the use of a slightly incorrect form of surface gravity anomalies in the computations.

For reasons which are not entirely clear it has been found that dynamically derived station positions of different investigators can have small systematic differences in the X, Y, Z values on the order of 5 meters in magnitude. It should be noted here that the GSFC long arc solution station coordinates have been modified to include a 10 meter correction to the Z values to account for a shift of the coordinate system from the center of mass. Because the tracking stations used in deriving the systemtic effects given in tables 1 through 3 are not uniformly distributed over the earth's surface one could anticipate the possibility of contributions of a few meters due to the failure of the origin of the coordinate system to coincide with the center of mass of the

earth. Of course the chosen value of a_e = 6378.142 kms could still be in error by 1 to 3 meters.

The causes of the small differences noted in Tables 1 through 3 for the gravimetric geoid heights and those obtained from the dynamic station positions of most investigators are no doubt the result of some combination of the above four error sources together with a large number of other small error contributors. However it should be noted that the above comparisons give no information concerning possible errors caused by the adopted values of $k M_{a+e}$ and $k M_e$. It seems unlikely these would exceed two to three meters.

4. PRESENTATION AND EVALUATION OF RESULTS

A detailed gravimetric geoid, a detailed gravimetric geoid – satellite geoid, and a satellite geoid for North America, the North Atlantic, Eurasia, and Australia were computed. These are presented as Figures 8 through 16.

To evaluate the precision of the detailed geoid for the areas computed, a number of comparisons were made. The first comparison was made with the astrogeodetic geoid data of Rice (1970) for the United States. Before any comparisons could be made, Rice's data were transformed from the North American Datum (NAD) to the geocentric coordinate system using several transformation sets. Table 4 presents the differences between Rice's astrogeodetic geoid and the gravimetric geoid using five different sets of translation elements after removing the mean differences. In all cases, the rms differences are on the order of 2 meters or less.

In Europe a comparison was made with Bomford's (1971) astrogeodetic geoid map. Bomford's astrogeodetic geoid values were first transformed from the European datum to the geocentric system using GSFC Long Arc Solution transformation sets of $\Delta X = 89$, $\Delta Y = 120$, and $\Delta Z = 118$ meters. It should be noted that these are mean translational values that do not incorporate the tilt of the European datum with respect

to the geocentric system. The comparisons were made along profile lines at latitudes 44°, 48° N and longitude 9° E. The comparisons of the transformed astrogeodetic geoid and the gravimetric geoid along the latitude profiles show an east-west rotation in the European datum which is not shown in the longitude profile (see Figures 1 through 3). This rotation was on the average equal to about 1.7 arc seconds. Allowing for this rotation in the datum, the relative agreement between Bomford's astrogeodetic geoid and the gravimetric geoid is within the ±2 meter range.

Another comparison in Eurasia was made with Fischer's astrogeodetic geoid map. Table 5 shows a comparison of the detailed geoid and the results of Fischer (1968) for a traverse across Eurasia at 52° N latitude. Both sets of geoid heights are referred to an ellipsoid of a_e = 6378.142 km, Fischer's values being taken directly from her paper. The geoid heights agree very well if one assumes a systematic error of 13 meters for Fischer's values west of 50° E longitude and 21 meters for values east of 50° E longitude. The cause for this disagreement in Europe has not been explored but could easily arise from a 10 to 15 meters error in the values used by Fischer to transform to the geocentric system as well as by the fact that she used only translation parameters to transform to the geocentric system while our investigations show that a rotation is also required. The change of 8 meters in the systematic difference at 50° E longitude could either arise from errors in connection of the Pulkova datum to the European datum or from systematic errors in the geophysically predicted free air anomalies used for gravimetric geoid computations in this area.

In Australia comparisons were made with Mather's Astrogeodetic geoid (1971). Again, before any comparisons were made, Mather's astrogeodetic data were transformed to the geocentric system using Mather's transformation values of $\Delta X = -125$, $\Delta Y = -30$, $\Delta Z = 111$ meters. The comparisons were made along profile lines at latitudes 22°, 26°, and 30° S (Figures 4 through 6). The comparisons along these latitude profiles

show an east-west rotation in the Australia datum approximately equal to 0.5 arc seconds. This rotation has also been noted by Mather (1971). Allowing for this rotation in the datum the relative agreement between Mather's astrogeodetic geoid and the gravimetric geoid is within ±2 meters.

5. COMPARISON OF SATELLITE GEOID WITH DETAILED GEOID

It is of interest to compare the satellite derived and detailed geoid maps. This comparison has been performed in two ways; by comparing latitude profiles and also by plotting contour maps of the differences.

Figure 7 presents a profile drawn across approximately one-half the globe at latitude 35°N comparing the detailed gravimetric geoid and the SAO '69 satellite geoid. The SAO '69 satellite geoid was referenced to $a_e = 6378.142$ km and a kM value of 3.986012×10^{20} cm^3/sec^2. Several conclusions result from the study of this profile:

1) The existence of a steep gradient in the North Atlantic.

2) There is no indication on the profile of any major tilt in North America.

3) There is an east-west tilt in the geoid of Europe.

4) In general the satellite geoid differs at most by approximately 10 m from the gravimetric geoid.

Figures 9, 12, and 15 present contour maps of the differences between the detailed geoid and the satellite geoid for Australia, North America, the North Atlantic and Eurasia. In Australia the most prominent differences occur in areas east of 135° longitude where they range between 10 and 20 meters. These large differences are attributed to the dominance of the eastern mountain ranges which adjoin relatively flat plains on the west and a shallow continental slope on the east. Table 6a lists coordinates for twelve locations in North America and the North Atlantic where the difference is in excess of 8 meters. The

difference of -12 meters over the Puerto Rican Trench was not unexpected since the gravity gradient is large over a small region. Other areas, for example; 42°N, 72°W and 49°N, 61°W where the differences are 14 meters and 10 meters respectively, may indicate broad shallow features to which satellites are not sensitive. Table 6b contains thirteen locations in Eurasia where the geoid height difference is larger than 8 meters. The relatively poorer agreement in Eurasia between the detailed geoid and the satellite geoid can probably be attributed to the following; 1) Variations of gravity are greater in Eurasia than in North America and are thus more difficult to detect from satellite motion, and 2) The surface gravity data in Asia is less accurate than that in Western Europe or North America.

6. CONCLUSIONS

The detailed gravimetric geoids presented here have a precision of ±2 meters over the continents and 5 to 7 meters where data was sparse.

The use of a consistent set of parameters has removed the systematic errors inherent in previously computed gravimetric geoids and hence reference these geoids to an absolute datum.

One question that might have been answered in this study concerns the possible existence of a rotation in different major datums. From this study there seems to be no conclusive evidence of a rotation in the North American datum but a rotation, which is prominent along the East-West profile does exist in the European and Australian datums. This rotation could be attributed to long wavelength errors in the satellite derived gravity model, a rotation of the astrogeodetic geoid or a combination of both.

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Table 1

GSFC Long-Arc Solution*/Gravimetric Geoid Comparison (meters)

(1) Station No.	(2) Station Name	(3) GSFC Long-Arc Geoid Height[a]	(4) Gravimetric Geoid Height	(5) ③ - ④	(6) ⑤ +7m[b]	(7) ⑤ +3m[c]
United States						
1021	Blossom Pt.	-41	-33	-8	-1	-5
1022	Ft. Myers	-31	-28	-3	4	0
1030	Goldstone	-35	-33	-2	5	1
1034	E. Grand Fks.	-30	-25	-5	2	-2
1042	Rosman	-41	-30	-11	-4	-8
7036	Edinburg	-36	-24	-12	-5	-9
7037	Columbia	-42	-32	-10	-3	-7
7045	Denver	-26	-19	-7	0	-4
7050	Greenbelt	-40	-32	-8	-1	-5
7072	Jupiter	-35	-32	-3	4	0
7075	Sudbury	-42	-37	-5	2	-2
Caribbean					⑤ - 1m	
7039	Bermuda	-41	-40	-1	-2	2
7040	San Juan	-53	-50	-3	-4	0
7076	Jamaica	-26	-31	5	4	8
Europe					⑤ + 0m	
1035	Winkfield	43	43	0	0	3
8009	Delft	40	41	-1	-1	2
8010	Zimmerwald	50	47	3	3	6
8015	Haute Provence	54	49	5	5	8
8019	Nice	47	48	-1	-1	2
9004	San Fernando	47	45	2	2	5
9091	Dionysos	41	42	-1	-1	2
9115	Oslo	40	37	3	3	6
9432	Uzhgorod	35	41	-6	-6	-3
Australia					⑤ + 6m	
1024	Woomera	7	12	-5	1	-2
1038	Orroral	24	25	-1	5	2
7054	Carnarvon	-23	-10	-13	-7	-10
9023	Is. Lagoon	7	12	-5	1	-2

a. Referenced to an ellipsoid with semimajor axis = 6378.142 km and a kM value of 3.986012×10^{20} cm^3/sec^2

b. Adjusted local difference

c. Adjusted global difference

*Marsh, Douglas and Klosko (1971)

Table 2

GSFC GEM 4 Solution*/Gravimetric Geoid Comparison (meters)

(1) Station No.	(2) Station Name	(3) GEM 4 Geoid Height[a]	(4) Gravimetric Geoid Height	(5) 3 - 4	(6) 5+0 m[b]	(7) 5+ 0 m[c]
United States						
1021	Blossom Pt.	-36	-32	-4	-4	-4
1022	Ft. Myers	-25	-28	3	3	3
1030	Goldstone	-28	-34	6	6	6
1034	E. Grand Fks.	-25	-25	0	0	0
1042	Rosman	-30	-30	0	0	0
7036	Edinburg	-26	-24	-2	-2	-2
7037	Columbia	-33	-32	-1	-1	-1
7045	Denver	-20	-19	-1	-1	-1
7072	Jupiter	-35	-32	-3	-3	-3
7075	Sudbury	-34	-35	1	1	1
Carribean					5 + 9m	
7039	Bermuda	-36	-40	4	-5	4
7040	San Juan	-45	-50	5	-4	5
7076	Jamaica	-13	-31	18	9	18
Europe					5 + 3m	
1035	Winkfield	49	43	6	9	6
9004	San Fernando	43	45	-2	1	-2
9091	Dionysos	28	42	-14	-11	-14
Australia					5 + 4m	
1024	Woomera	12	12	0	4	0
1038	Orroral	25	25	0	4	0
7054	Carnarvon	-25	-10	-15	-11	-15
9023	Is. Lagoon	11	12	-1	3	-1

a. Referenced to an ellipsoid with semimajor axis = 6378.142 km and a kM value of 3.986012×10^{20} cm^3/sec^2

b. Adjusted local difference

c. Adjusted global difference

*Lerch, Wagner, Smith, Sandson, Brownd and Richardson, (1972)

Table 3

SAO '69 Solution*/Gravimetric Geoid Comparison (meters)

(1) Station No	(2) Station Name	(3) SAO Geoid Height[a]	(4) Gravimetric Geoid Height	(5) ③ - ④	(6) ⑤ +3m[b]	(7) ⑤ +0m[c]
United States						
1021	Blossom Pt.	-34	-32	-2	1	-2
1034	E. Grand Fks	-24	-25	1	4	1
1042	Rosman	-41	-30	-11	-8	-11
7037	Columbia	-33	-32	-1	2	-1
7045	Denver	-12	-19	7	10	7
7050	Greenbelt	-34	-32	-2	1	-2
7075	Sudbury	-44	-35	-9	-6	-9
9001	Organ Pass	-28	-22	-6	3	6
9010	Jupiter	-32	-32	0	3	0
9021	Mt. Hopkins	-34	-28	-6	-3	-6
9050	Harvard	-38	-25	-13	-10	-13
9113	Edwards AFB	-30	-35	5	8	5
Caribbean					⑤ -15m	
7039	Bermuda	-28	-40	12	-3	12
7040	San Juan	-46	-50	4	-11	4
7076	Jamaica	-2	-31	29	14	29
Australia					⑤ + 1m	
9023	Is. Lagoon	12	12	0	1	0
9003	Woomera	10	12	-2	-1	-1

a. References to an ellipsoid with semimajor axis = 6378.142 km and a kM value of 3.986012×10^{20} cm^3/sec^2

b. Adjusted local difference

c. Adjusted global difference

*Gaposchkin and Lambeck (1970)

Table 4

Comparison of Detailed Gravimetric Geoid and Rice's Astrogeodetic Geoid Under Different Assumptions for Transforming Astrogeodetic Data

Latitude, N	Longitude, W	1	2	3	4	5
34° 58' 03"0	120° 38' 05"5	2	-1	2	3	1
35 00 38.0	119 00 48.0	2	0	4	3	1
38 47 23.1	121 52 15.6	0	-3	1	1	-1
35 02 36.1	106 30 24.1	1	1	2	1	1
32 13 14.7	106 29 41.6	3	2	3	3	2
32 00 00.6	103 16 07.2	0	0	0	0	-1
30 59 40.0	098 05 50.5	-2	-1	-1	-2	-2
30 36 26.5	091 23 18.1	-1	1	-1	-1	-1
29 38 10.8	091 06 49.3	-1	1	0	-1	-1
30 59 25.5	089 34 29.5	-2	0	-2	-2	-2
28 29 28.6	080 33 35.6	3	6	3	3	3
30 36 53.3	081 42 14.8	2	5	2	3	2
39 28 18.9	076 05 15.2	-3	-1	-4	-3	-3
34 59 44.0	076 59 11.7	-3	1	-3	-2	-2
33 28 42.4	091 00 08.5	-1	1	-1	-1	-1
33 34 48.5	092 50 07.2	0	2	1	0	0
34 56 47.0	093 24 18.3	-2	-1	-2	-2	-2
37 38 08.4	094 35 46.8	-2	0	-1	-1	-2
35 03 04.0	097 56 52.6	0	0	0	0	0
39 13 26.7	098 32 30.5	0	0	0	-1	0
43 37 10.7	096 17 52.3	0	1	0	-1	0
35 06 16.2	103 19 55.0	2	2	2	2	1
34 56 32.8	096 24 55.3	0	1	0	0	0
44 43 46.0	105 25 50.7	2	1	2	1	1
36 47 44.2	103 11 48.5	1	1	1	1	0
38 50 40.6	102 48 46.8	0	0	1	0	0
48 06 18.6	102 21 09.7	-2	-2	-2	-2	-3
46 44 47.4	102 15 13.4	-2	-2	-2	-2	-2
45 12 45.7	102 09 14.1	0	-2	-2	-2	-2
46 21 53.1	108 59 07.3	1	0	1	1	0
31 03 07.3	102 56 05.8	0	0	0	0	-1
41 30 419.	097 37 23.4	-1	0	-1	-1	-1
30 48 49.8	093 12 26.9	-3	-1	0	-4	0
47 50 28.9	110 00 46.4	1	0	1	1	0

1 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using GSFC Long Arc (Marsh, et al., 1971) translation values of Δx = -25.1, Δy = 162.9, and Δz = 172.5 meters; mean scale difference = 16m.

2 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using Fischer's (1968) translation values of Δx = -18, Δy = 145, and Δz = 183; mean scale difference = 2m.

3 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using SAO Standard Earth '66 translation values of Δx = -30, Δy = 152, and Δz = 176 (Lundquist and Veis, 1967); mean scale difference = 5m.

4 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using SAO's transformation values of Δx = -25.8, Δy = 168.1, and Δz = 167 (K. Lambeck, 1971, personal communication); mean scale difference = 23m.

5 = Corrected difference between Rice's astrogeodetic geoid and the detailed gravimetric geoid using GSFC GEM 4 transformation values of Δx = -24, Δy = 153, and Δz = 181 (Lerch et al., 1971); mean scale difference = 3m.

Table 5

Fischer's Astrogeodetic Geoid/Gravimetric Geoid Comparison for Eurasia at 52 °N Latitude (meters)

Long. East	① Fisher Geoid	② Grav. Geoid	② - ①	② - ① -13m	② - ① -21m
4°	34	+41	+7		
8	32	+43	+11	-2	
12	29	+43	+14	+1	
16	23	+39	+16	+3	
20	17	+32	+15	+2	
24	13	+36	+13	0	
28	10	+21	+11	-2	
32	3	+16	+12	-1	
36	-5	+10	+15	+2	
40	-11	+6	+17	+4	
44	-17	0	+17	+4	
48	-23	-6	+17	+4	
52	-31	-9	+22		+1
56	-36	-14	+22		+1
60	-40	-17	+23		+2
64	-46	-24	+22		+1
68	-50	-29	+21		0
72	-58	-36	+22		+1
76	-62	-42	+20		-1
80	-65	-46	+19		-2
84	-65	-47	+18		-3
88	-65	-44	+21		0
92	-66	-47	+19		-2
96	-68	-48	+20		-1
100	-65	-49	+16		-5

Table 6-a.
(Detailed Gravimetric Geoid – Satellite Geoid)
North America

Location	Δ h (meters)
1. 47° N, 115° W	-8
2. 27° N, 107° W	8
3. 24° N, 95° W	-10
4. 55° N, 83° W	-10
5. 42° N, 72° W	14
6. 28° N, 71° W	-8
7. 17° N, 61° W	-12
8. 49° N, 61° W	-10
9. 43° N, 49° W	12
10. 22° N, 52° W	-8
11. 22° N, 30° W	12
12. 38° W, 20° W	14

Table 6-b.
(Detailed Gravimetric Geoid – Satellite Geoid)
Eurasia

Location	Δ h (meters)
1. 46° N, 5° W	-16
2. 39° N, 3° W	-14
3. 41° N, 16° E	18
4. 41° N, 21° E	18
5. 54° N, 28° E	-13
6. 44° N, 37° E	-18
7. 31° N, 72° E	-10
8. 34° N, 79° E	14
9. 46° N, 86° E	-14
10. 26° N, 85° E	-16
11. 19° N, 88° E	12
12. 44° N, 103° E	16
13. 62° N, 105° E	24

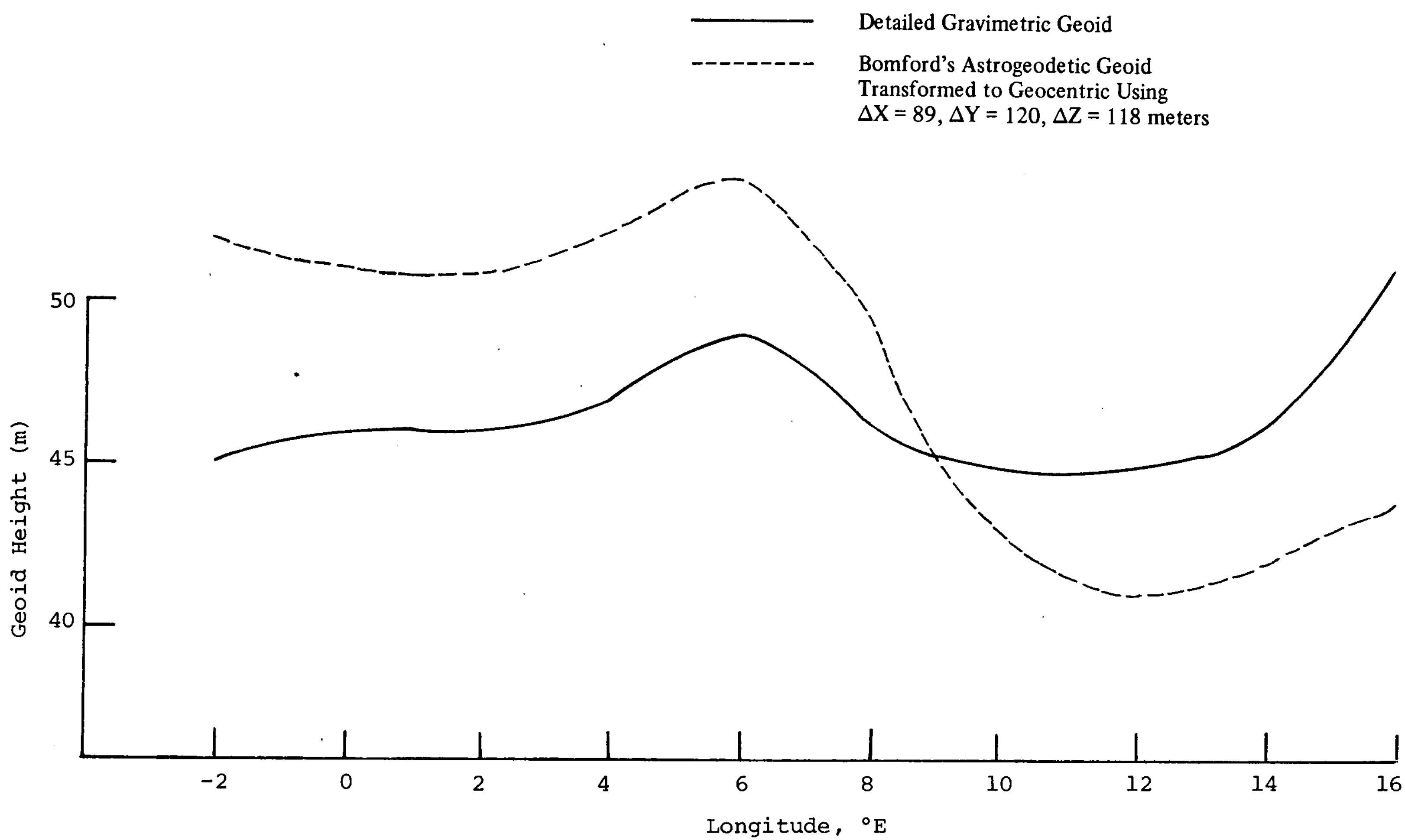

Figure 1 – Detailed Gravimetric Geoid and Bomford's Transformed Astrogeodetic Data at 44°N (Adjusted for 10m Scale Difference)

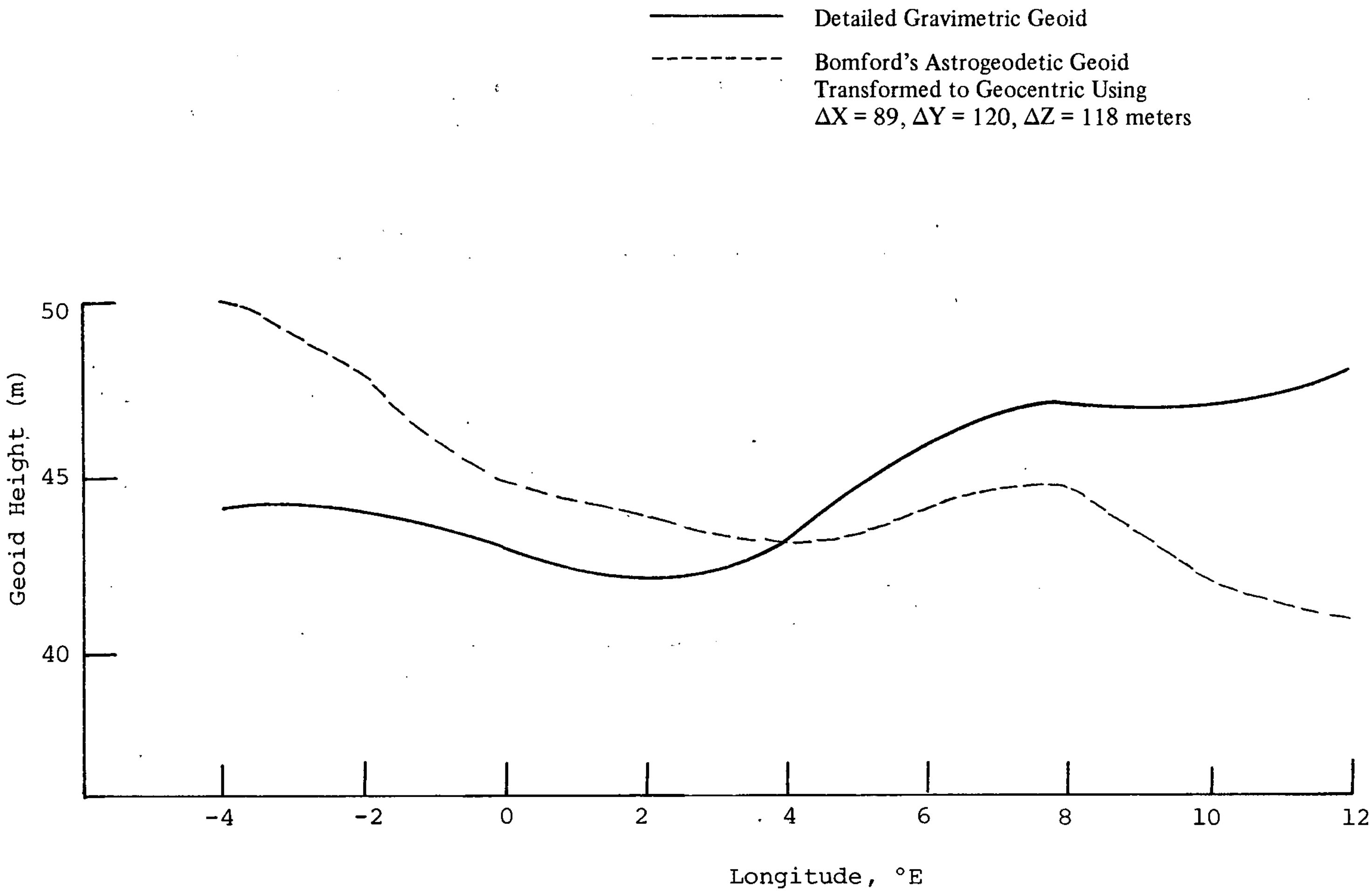

Figure 2 – Detailed Gravimetric Geoid and Bomford's Transformed Astrogeodetic Geoid at Latitude 48°N (Adjusted for 10m Scale Difference)

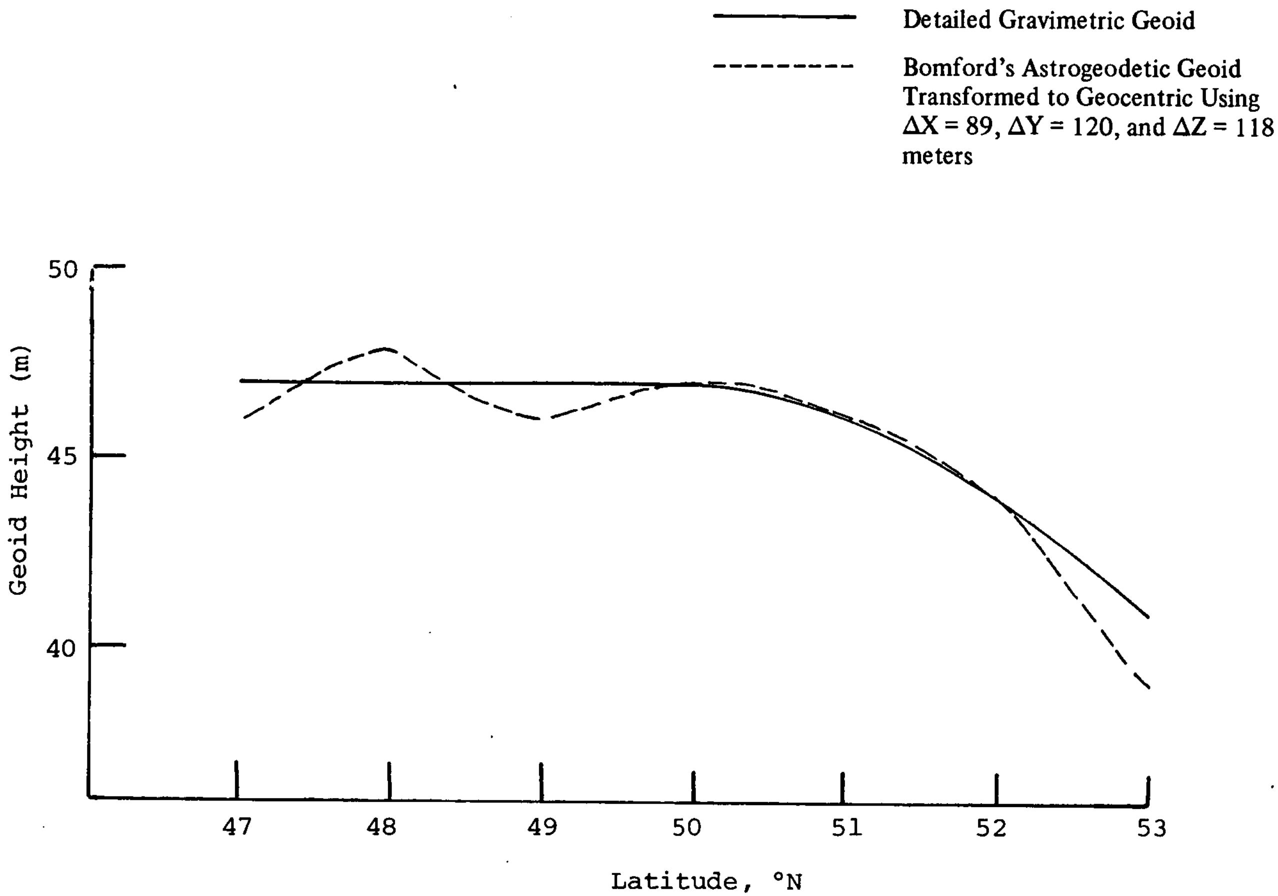

Figure 3 – Detailed Gravimetric Geoid and Bomford's Transformed Astrogeodetic Geoid at 9°E (Adjusted for 10m Scale Difference)

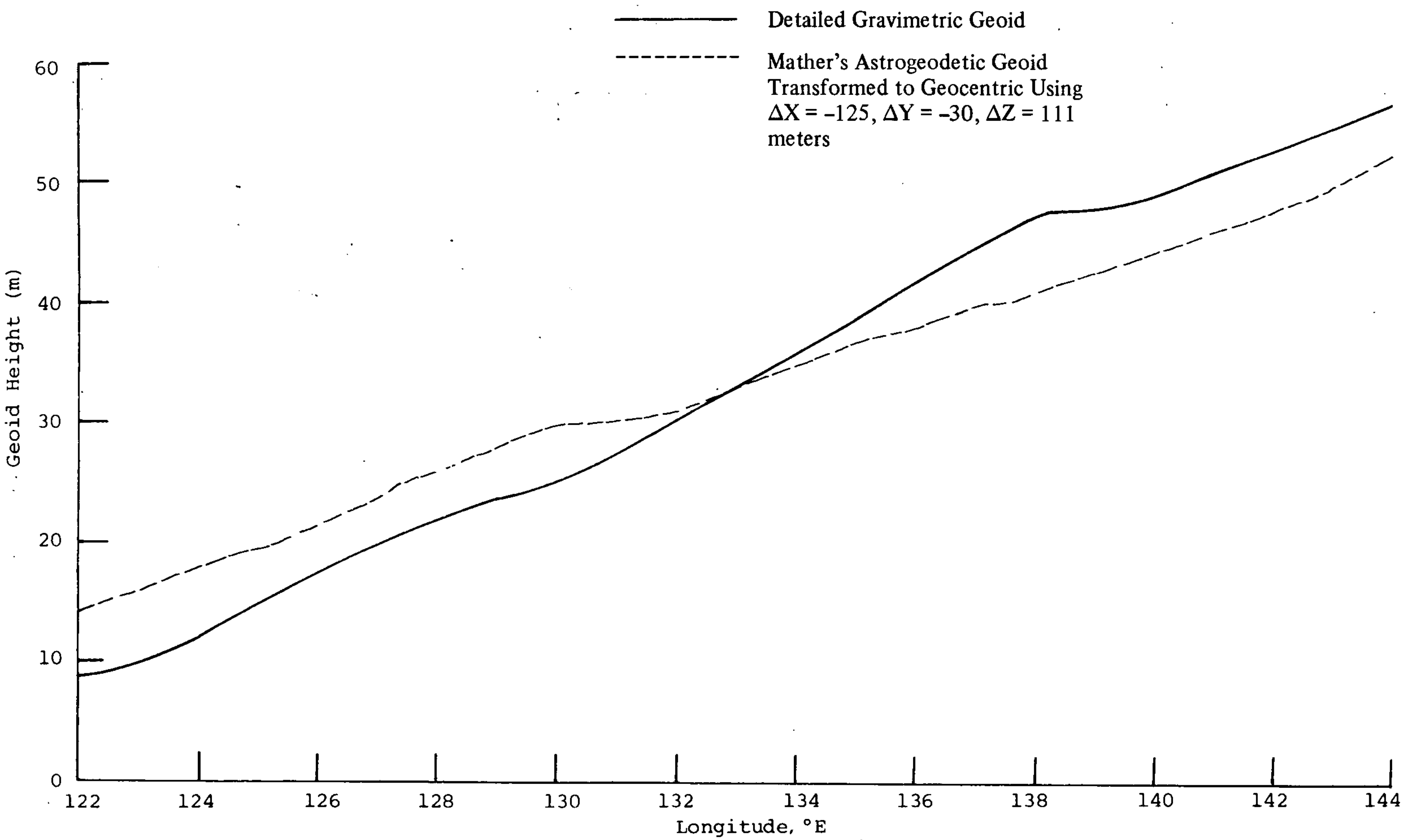

Figure 4 - Detailed Gravimetric Geoid and Mather's Astrogeodetic Data at 22° S (Adjusted for 12 m scale difference)

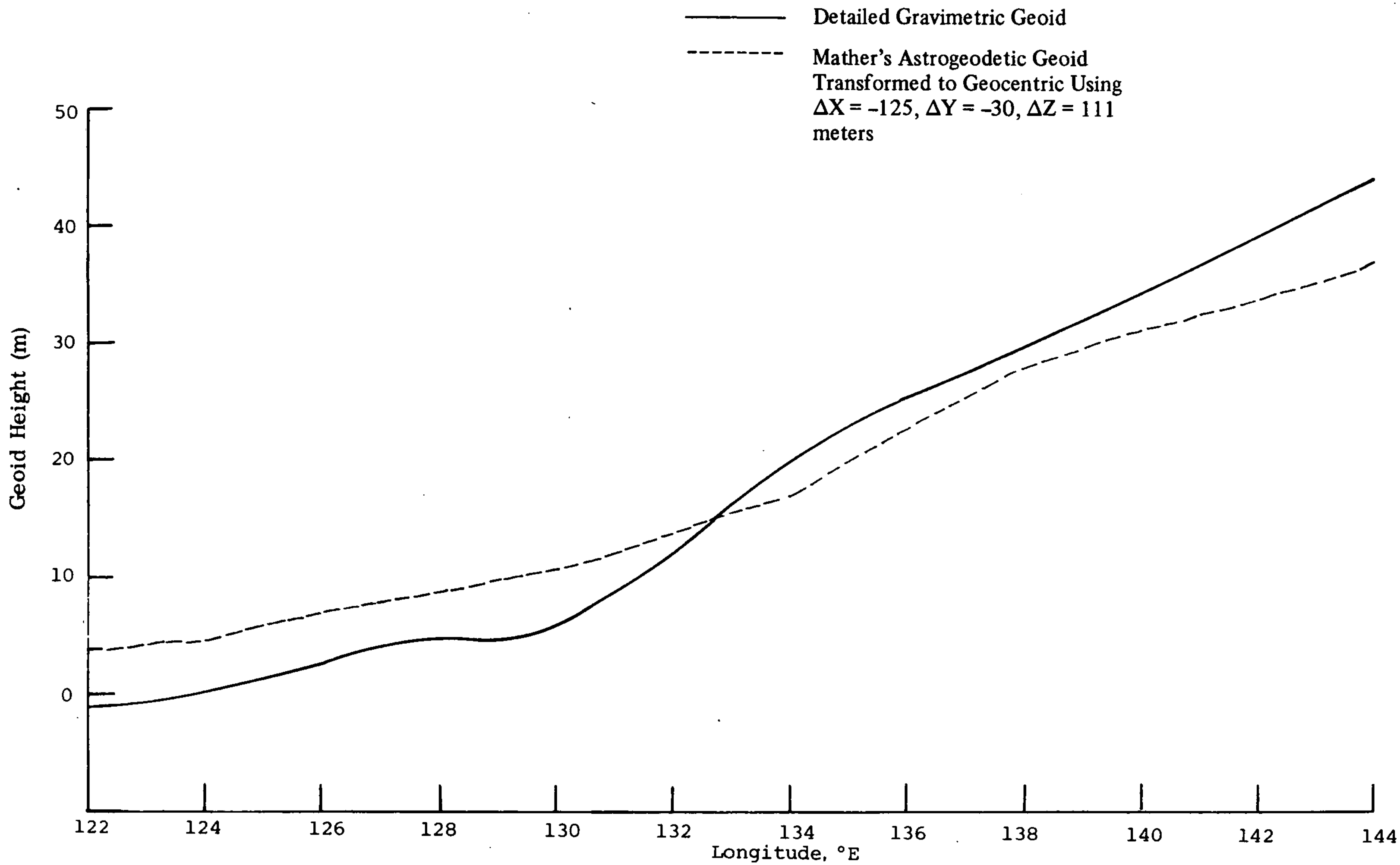

Figure 5 - Detailed Gravimetric Geoid and Mather's Astrogeodetic Data at 26° S (Adjusted for 12 m scale difference)

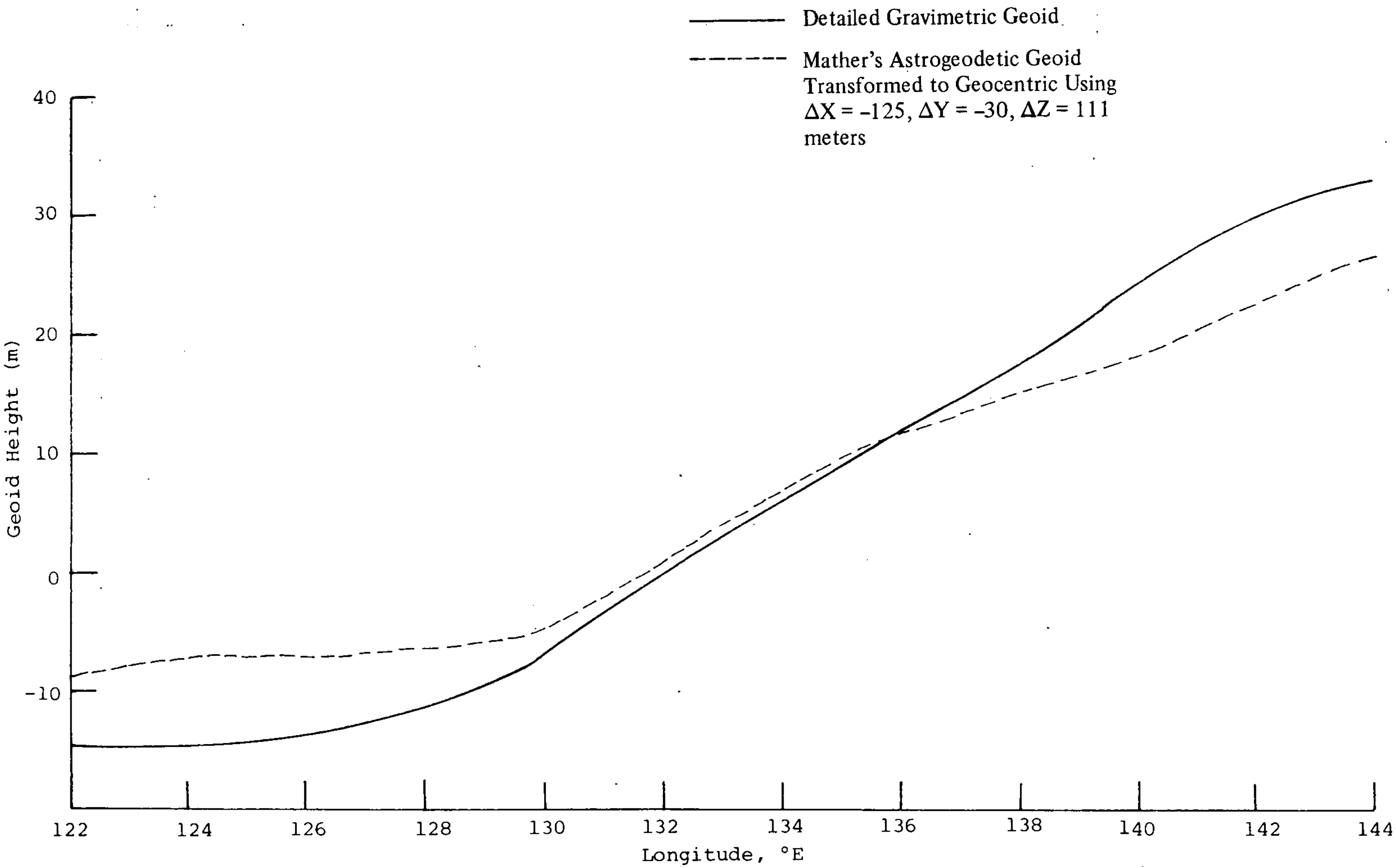

Figure 6 - Detailed Gravimetric Geoid and Mather's Astrogeodetic Data at 30° S
(Adjusted for 12 m scale difference)

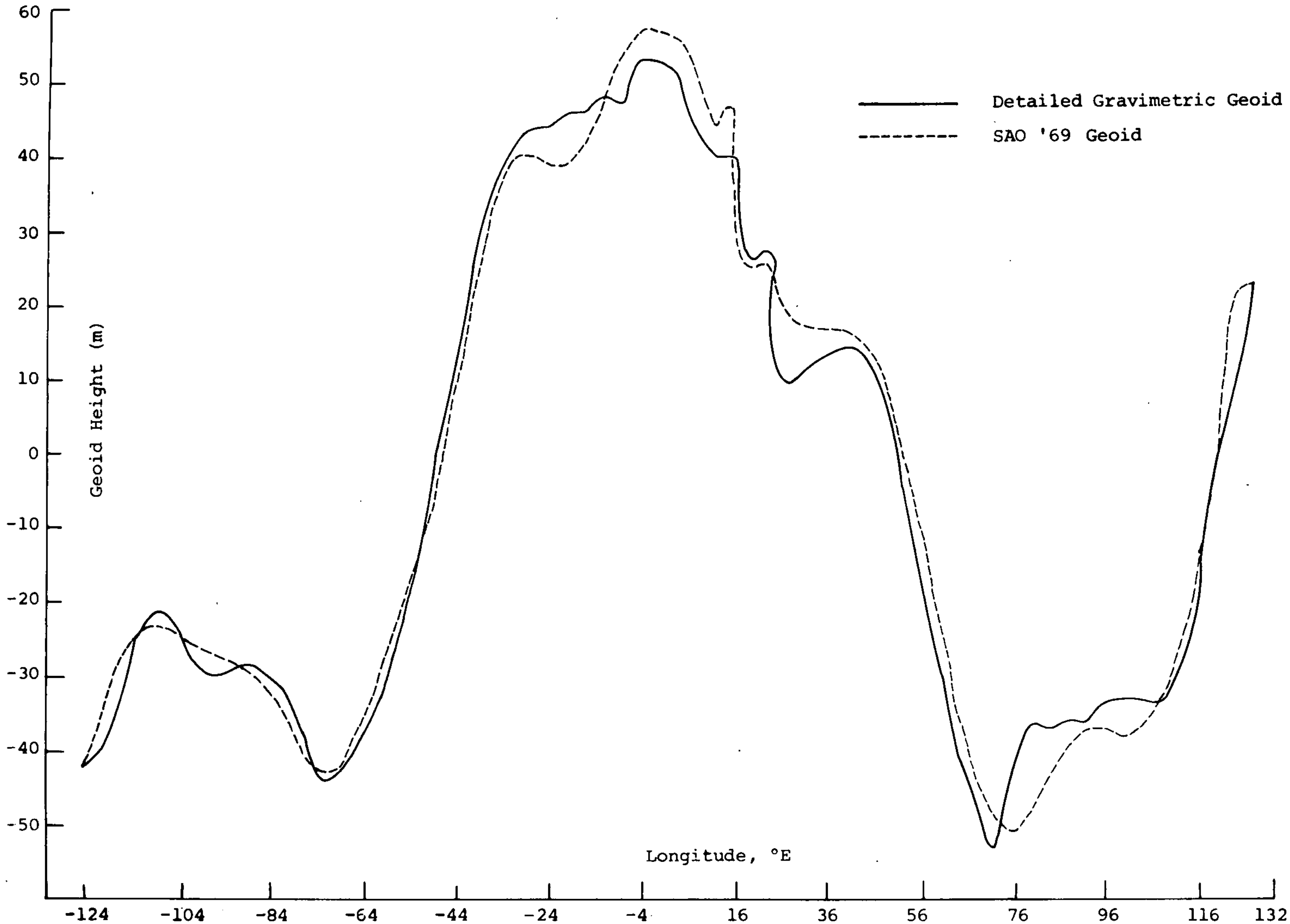

Figure 7 – Detailed Gravimetric Geoid and SAO '69 Geoid at Latitude 35°N

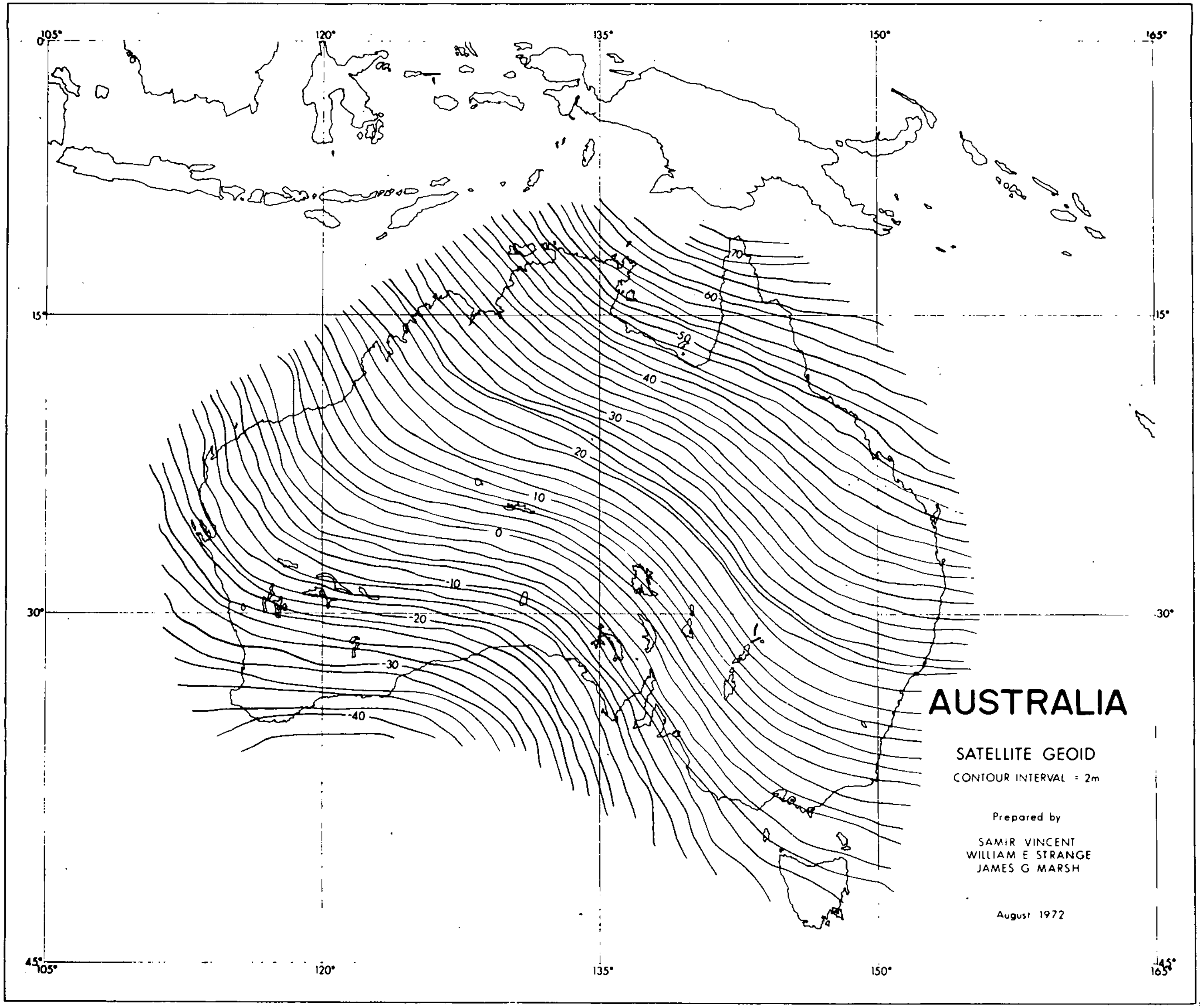

Figure 8 – Australia Satellite Geoid

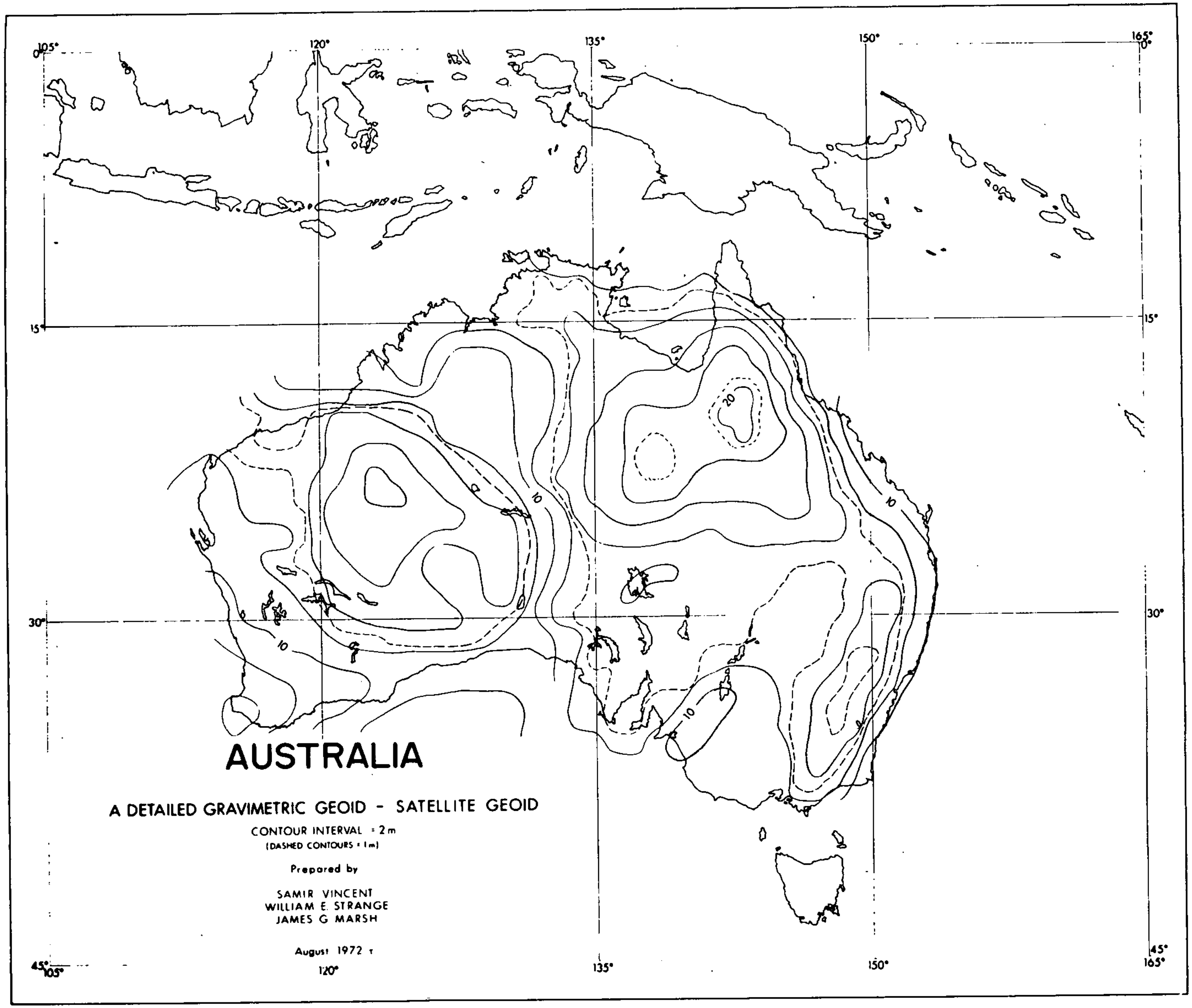

Figure 9 – Australia – A Detailed Gravimetric Geoid-Satellite Geoid

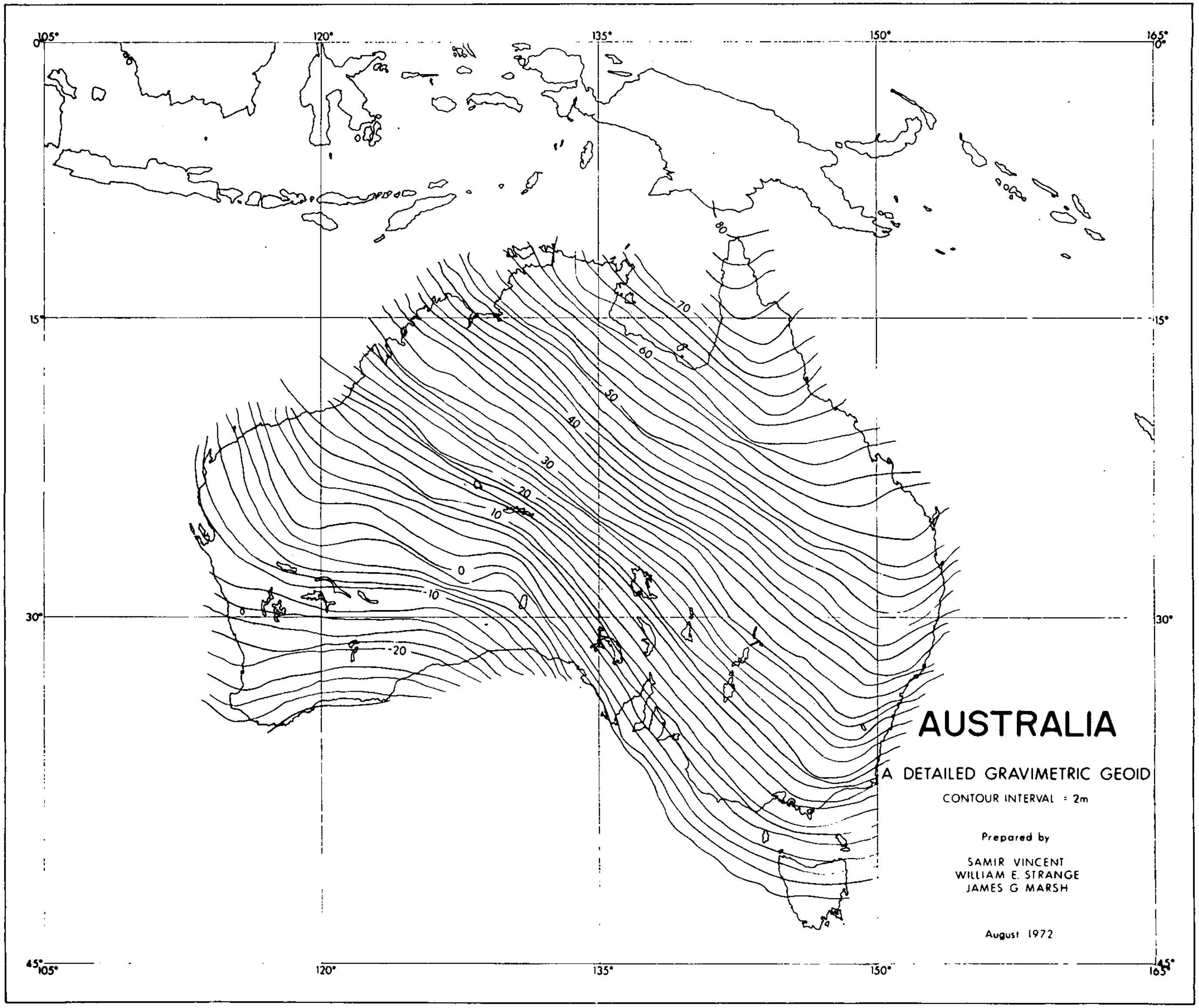

Figure 10 – Australia – A Detailed Gravimetric Geoid

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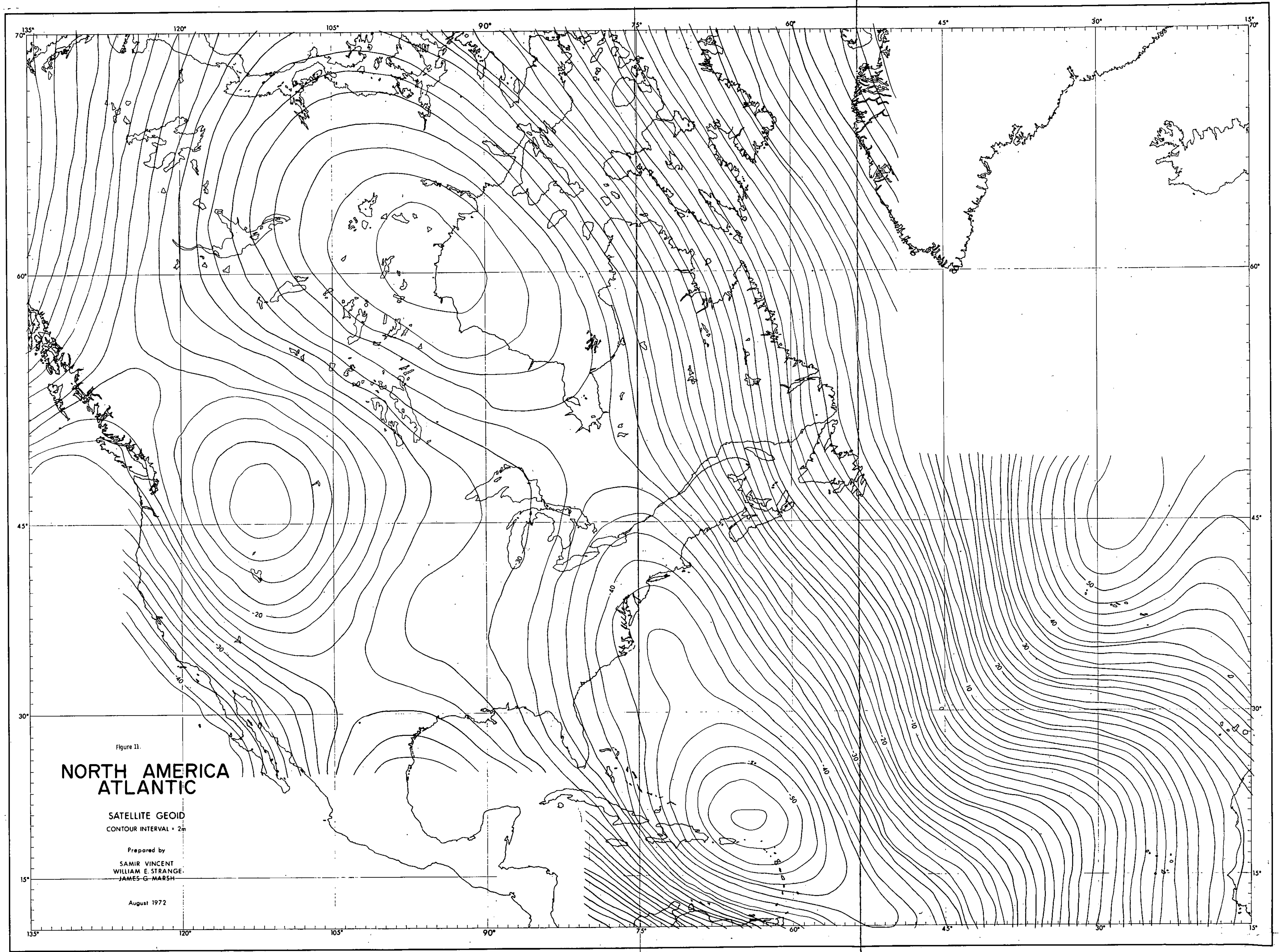


Figure 11.
NORTH AMERICA
ATLANTIC
SATELLITE GEOID
CONTOUR INTERVAL = 2m
Prepared by
SAMIR VINCENT
WILLIAM E. STRANGE
JAMES G. MARSH
August 1972

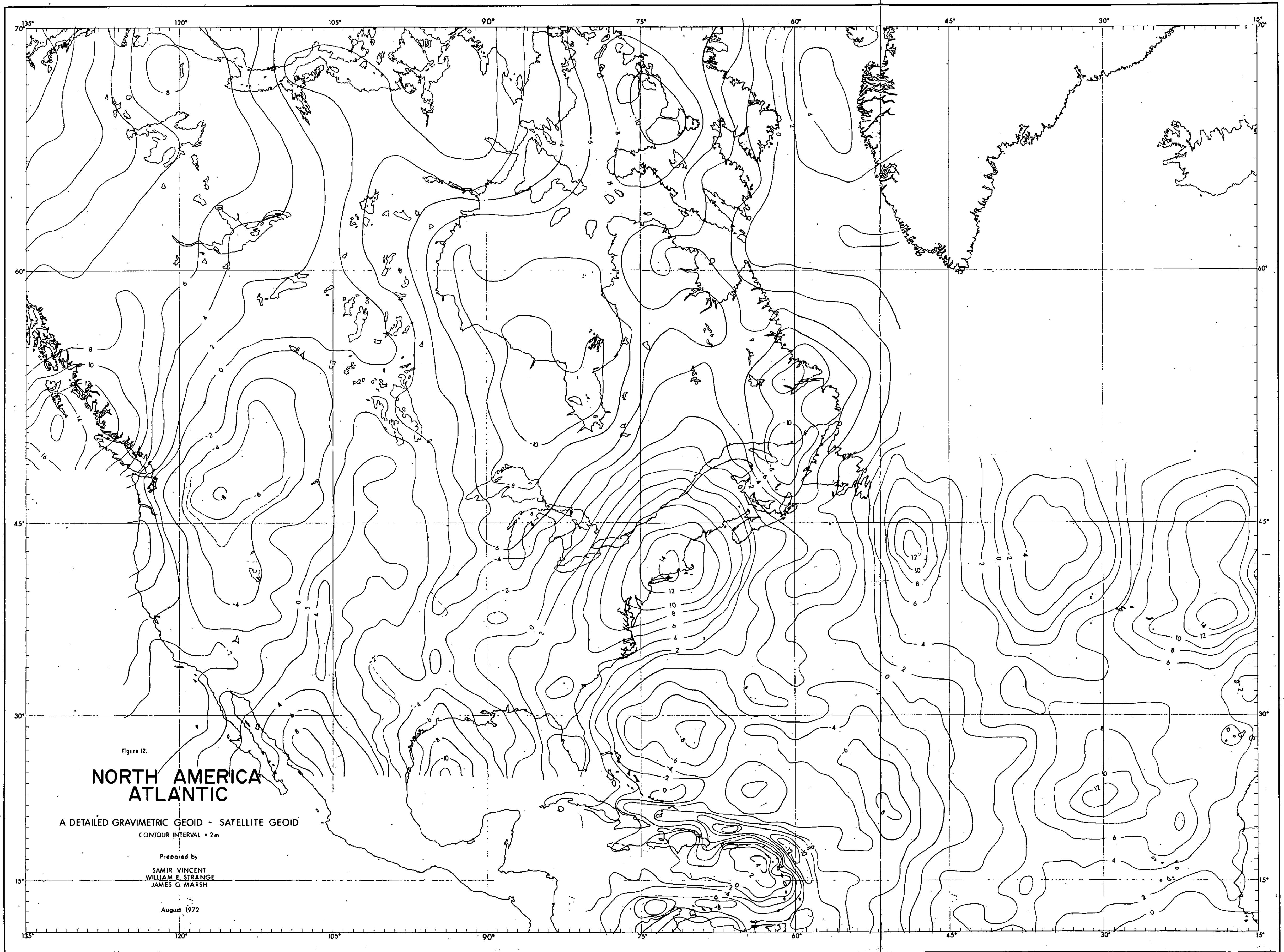


Figure 12.

NORTH AMERICA ATLANTIC

A DETAILED GRAVIMETRIC GEOID - SATELLITE GEOID

CONTOUR INTERVAL = 2m

Prepared by

SAMIR VINCENT
WILLIAM E. STRANGE
JAMES G. MARSH

August 1972

FOLDOUT FRAME 1

FOLDOUT FRAME 2

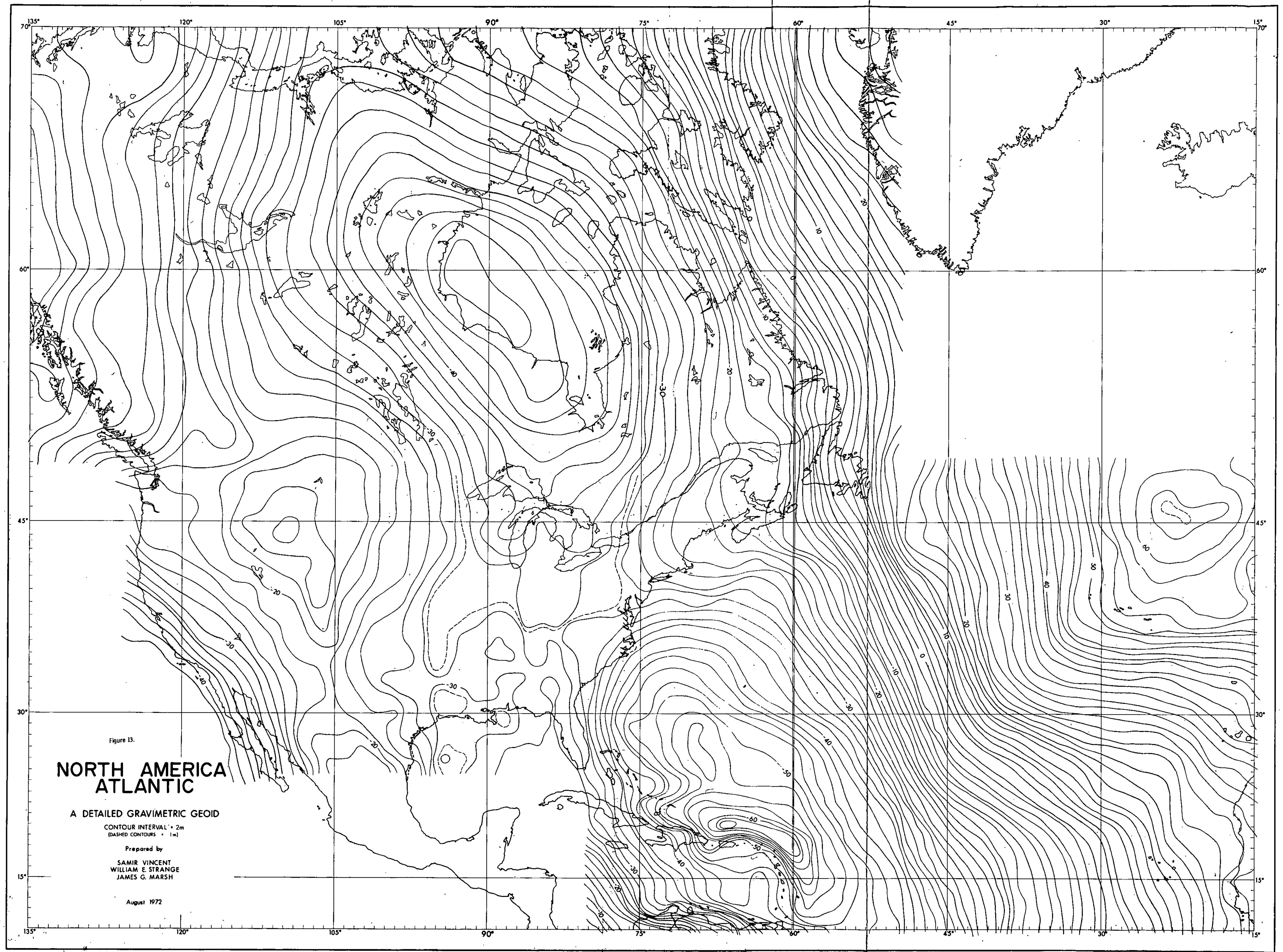


Figure 13.

NORTH AMERICA
ATLANTIC

A DETAILED GRAVIMETRIC GEOID

CONTOUR INTERVAL = 2m
(DASHED CONTOURS = 1m)

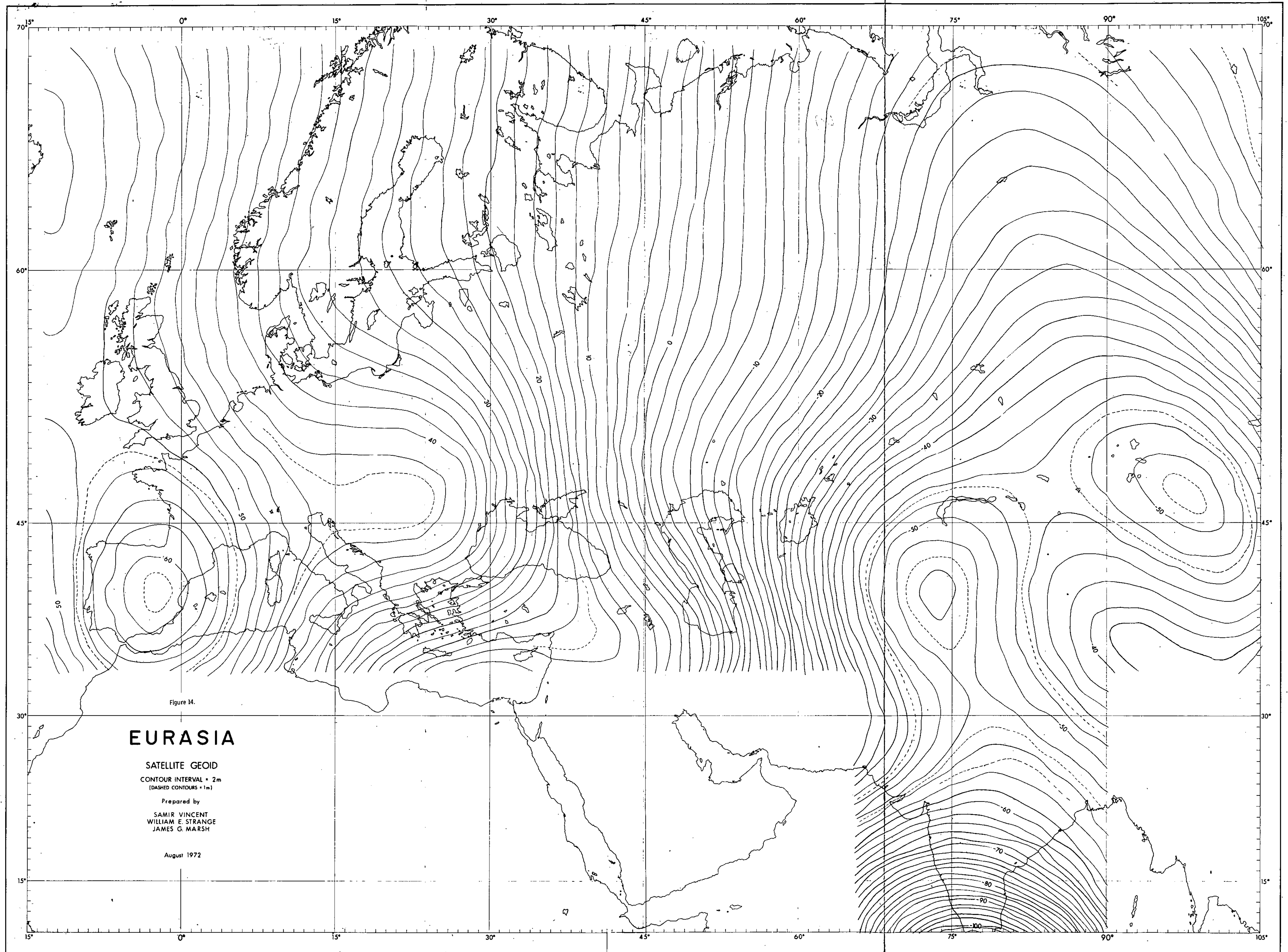
Prepared by

SAMIR VINCENT
WILLIAM E. STRANGE
JAMES G. MARSH

August 1972

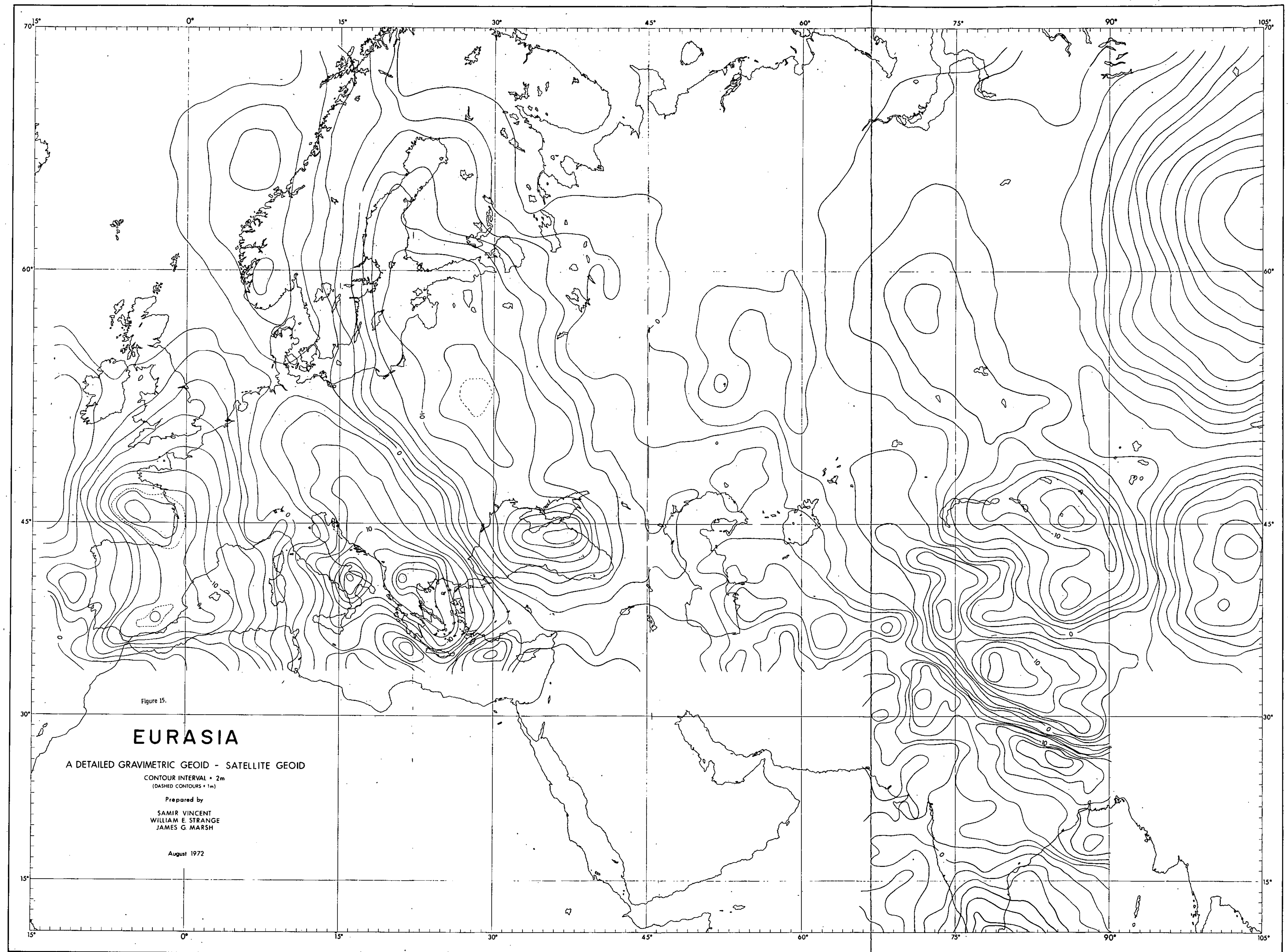
FOLDOUT FRAME 1

FOLDOUT FRAME 2



FOLDOUT FRAME 1

FOLDOUT FRAME 2



FOLDOUT FRAME 1

FOLDOUT FRAME 2

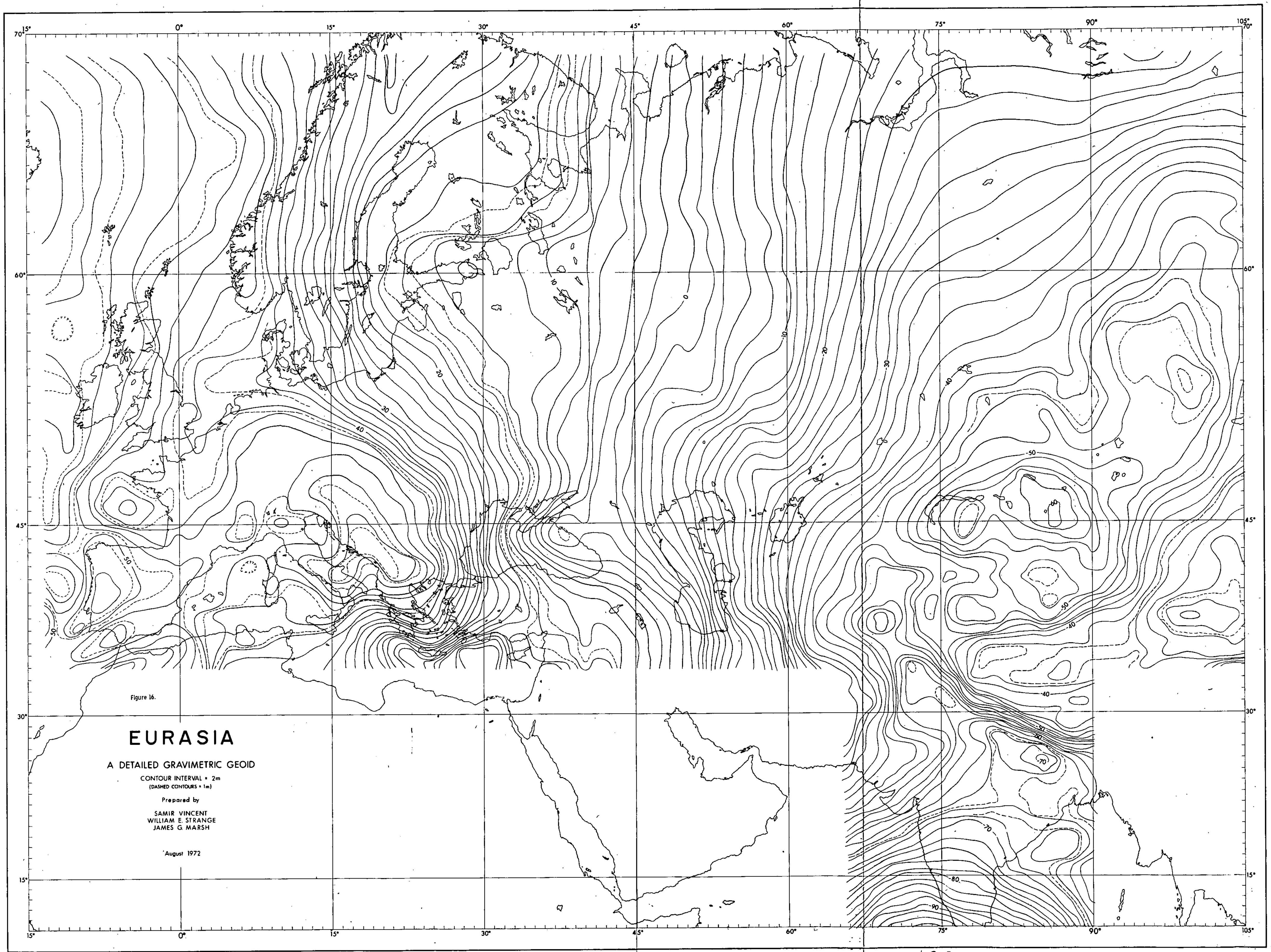


Figure 16.
EURASIA
A DETAILED GRAVIMETRIC GEOID
CONTOUR INTERVAL = 2m
(DASHED CONTOURS = 1m)
Prepared by
SAMIR VINCENT
WILLIAM E. STRANGE
JAMES G. MARSH
August 1972